Parallel Moves in Hanoi Games on Graphs: Every Parallel Hanoi Graph is a Sequential Hanoi Graph

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We define two different models of parallel moves for Hanoi games, the one-step and the two-step models. Every sequential move is a one-step parallel move, and every one-step parallel move is a two-step parallel move. However, certain parallel moves cannot be replaced by sequential moves. Parallel moves may significantly reduce the number of moves required to move n disks from the starting peg (S) to the destination peg (D): The graph ({S,A,D}, {(S,A), (A,S), (A,D), ((A,D)}) requires 3^n-1 or $O(3^n)$ sequential moves but using two-step parallel moves, $2 \cdot 3^{n/2}$-2 or $O(3^{n/2})$ moves are sufficient. Several decades ago, the Towers of Hanoi game was generalized to be played on arbitrary directed graphs. All finite directed graphs that permit moving sequentially any number of disks from S to D were characterized. The fact that parallel moves cannot be always replaced by sequential moves raises the question whether the characterization (for sequential moves) carries over to parallel moves. We show that every graph that permits the move of an arbitrary number of disks from S to D using parallel moves is also a graph that permits this using sequential moves.

Keywords: Towers of Hanoi, parallel moves, Hanoi graphs.
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Scheduling Request

Please schedule this talk on Tuesday (March 8, 2016) or Wednesday (March 9, 2016).

Thank you!

Ernst L. Leiss